

A Two-Factor
Markov Functional Model
for pricing interest rate derivatives

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Diploma Thesis

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Abstract

The present diploma thesis delves into a new model class for pricing interest rate derivatives, namely the Markov Functional Models (MFM). First, the necessary fundamentals of stochastic calculus and derivative pricing are laid down. A multifactor interest rate model (IRM), the LIBOR Market Model (LMM), is discussed and the main subject, the MFM, is described in detail. A one-factor and a two-factor model will be explained and implemented with the forward London Inter-Bank Offered Rate (LIBOR) as modeling basis. Finally, the impact of different parameterizations of a two-factor LIBOR Markov Functional Model (LMFM) is analyzed by means of swaptions and options on a swap rate spread.

“When a tree is growing, some branches prosper straight out of the trunk, while others aim in life is to wait until they flourish high above. The lower boughs are awesome in strength, but usually less exposed. Others live in the dark, but build the firmness within the fortress of pure nature. Without sun, they live in harmony and in symbiosis. Even, when its life is over, the strength of its roots seems to rest everlasting.”¹

¹The author.

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